DYNAMIC LUMPED-ELEMENT MODELING AND SIMULATION OF A PERSON RIDING A BICYCLE

Part One: evaluating a bicycle's ride quality by its frequency response

© Osman Isvan; January, 2021

SUMMARY

The mechanical structure of a person riding a bicycle is represented by a dynamic lumped-element model characterized by its natural frequencies and mode shapes. Static loads are superimposed with dynamic loads with random excitations due to ground reaction forces when the bicycle is ridden on an uneven road surface. The structure reacts to these inputs according to its "frequency response", which typically serves as a low-pass filter that isolates the rider from road vibrations. The effective suspension compliance arises from the elastic properties of structural components such as tires, fork blades, "frame flex", muscles and tendons. To demonstrate the viability of dynamic modeling with lumped elements, I use a simplified model with only 3 degrees of freedom and simulate system response under steady state and transient conditions. I investigate the effects on vibration isolation, of several key parameters including tire pressure, frame compliance and riding position.

INTRODUCTION

When choosing a frame and fork for a road bike, geometric variables that affect fit and comfort must be reconciled with those that affect handling and performance. But once the geometry is dialed in, what are other important factors? Strength, lightness and stiffness must be among the top priorities. The benefits of strength and lightness are obvious. Stiffness is important too, because, as the forces due to pedaling, cornering and road vibrations try to interfere with ride quality, it is the frame's responsibility to keep the head tube (and hence the steering axis) in the plane of the rear wheel. As well, fork blades must be stiff enough to keep the steerer tube in the plane of the front wheel, and the front hub parallel to the handlebar. These requirements must be met to ensure proper handling. On the other hand, the frameset (i.e., frame and fork) must not be too stiff, because on rough roads it functions as suspension and isolates the rider from road vibrations. Not only does vibration isolation increase comfort, but the reduction in unsprung mass improves traction and control. In the first part of this two-part article, a dynamic simulation model is introduced, which may be useful for designing bicycle frames to improve *comfort*. The Second Part of the article will be devoted to the concept of *dynamic rolling resistance* and using the model to improve *performance*.

The requirement for a bicycle to present a stiff response to handling and pedaling inputs, but a compliant response to inputs arising from bumps on the road may appear to be antithetical at first, but there is empirical evidence that some bicycles exhibit this behavior to some degree. In a dynamic system, such behavior is possible if these two sets of inputs are differentiated by their *frequency content*. This article is about modeling the bicycle and the rider's body as an integrated dynamic structure that is characterized by *frequency-dependent* mass, stiffness and damping properties, otherwise known as *mechanical impedance*.

According to this model, the amplitude and mode shape of the deformations of the structure in response to an applied force depends on the location, direction and *frequency* of the force, and the unique set of natural frequencies and mode shapes that are intrinsic to the structure.

In a typical load case, multiple input forces are applied at the same time. The parameter values of the simulation model can be varied until the *frequency response* of the structure is optimized to improve vibration isolation and handling characteristics. One of the most elusive of these parameters is frame stiffness.

FRAME STIFFNESS

For efficient power transfer, in general the frame must be stiff in the direction of the pedal force, and for a comfortable ride, it must be compliant in the vertical direction. The challenge in meeting these requirements simultaneously is that peak pedal forces are almost always applied in a near-vertical direction. This creates a dilemma for static models in the development of bicycle frames. The proposed approach of dynamic modeling overcomes this dilemma to some degree, as we shall see, but we start by looking at the progression of prevailing concepts about frame stiffness.

Ultimate rigidity: In the early stages of the evolution of bicycles, limitations in available materials and manufacturing technologies made it virtually impossible to produce a sufficiently lightweight frame stiff enough to satisfy everyone. This limitation cultivated a mindset and culture where lightness and stiffness were ultimately desirable attributes. This “more is better” concept represents the *first stage* of understanding frame stiffness.

Stiffness in moderation: In the early 1980s, when thin-walled, large diameter tubing made of high-strength aluminum alloys began to be used in mass-produced bicycle frames, it became apparent that a lightweight bicycle can be “too stiff”. This paradigm shift led to a period of unprecedented reverence for bicycles made of titanium tubing. Eventually, the exceptional lightness and “tunability” of carbon fiber composites and fabrication methods found greater acceptance, but even today, in subjective evaluations of high-performance bicycles, especially those made of titanium or steel, the absence of rigidity is often described by positive attributes such as *liveliness* and *resilience*, rather than necessarily being considered a shortcoming. Similarly, response characteristics and road manners resulting from excessively stiff frames and forks are sometimes described as harsh, jarring, unforgiving or “dead”. It appears as though, somewhat paradoxically, we want the frame and fork to be rigid and flexible at the same time. To add to the confusion, there is the notion of *comfort*, which is sometimes associated with a material property called *damping*, a hallmark of carbon fiber; yet carbon fiber, especially in the most sought-after *high modulus* category, is extremely stiff. This leads to the *second stage* of our collective understanding of frame stiffness, which holds that the ideal frame should be sufficiently stiff, but with a measured amount of compliance. According to this theory, the key to successful engineering of bicycle frames hinges on dialing-in just the right amount of compliance in order to find the optimum balance between two opposing requirements: comfort on the one hand, and performance on the other. This more nuanced but still simplistic interpretation of frame stiffness can be illustrated with a one-dimensional visualization, as shown in Figure 1. According to this view, frame flex is required for comfort, at the expense of performance, and the primary objective of bicycle design is to find the sweet spot between “too flexible” and “too stiff” extremes. For the purpose of illustration, in Figure 1, Bike 2 represents the most successful optimization among the four examples, because it is closest to the sweet spot at the center of the axis, although performance-oriented buyers might opt for Bike 3 instead. On the other hand, Bike 1 may be acceptable only to those who don’t care about performance, and Bike 4 clearly misses the mark by being too stiff and therefore, necessarily uncomfortable. This interpretation represents the *second stage* of understanding frame stiffness.

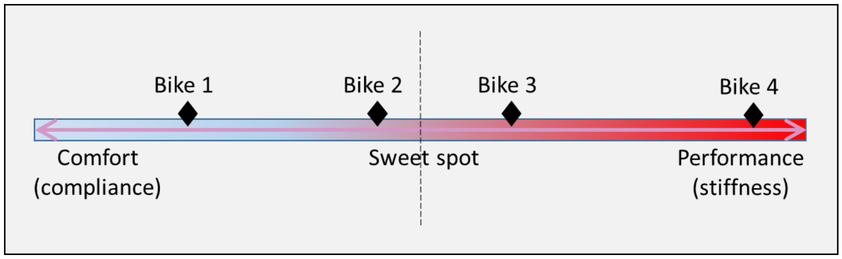


Figure 1 –One-dimensional visualization of frame stiffness

Directional stiffness: The inseparable association between ride comfort and frame compliance stems from a rather rudimentary understanding of mechanical structures. But once bicycle frames made of different materials and with different stiffness characteristics became generally available, it was discovered that frame flex comes in at least two varieties: vertical flex and lateral flex. (Flex is another term for compliance; it is the inverse of stiffness). This theory holds that vertical compliance provides comfort by absorbing road shock without significantly compromising performance, whereas lateral and torsional compliance are impediments to "efficient power transfer". To put this theory into practice, or at least to take advantage of its marketing potential, bicycle manufacturers, independent test labs, and even cycling magazines, developed special test equipment and methods to measure vertical, lateral and torsional frame deformations under specified test conditions. However, a general consensus has not been reached on how these test results might translate into real-world cycling performance or comfort. I believe that the primary reason for this lack of consensus is that the relevant factors are inherently dynamic, so they cannot be quantified by tests performed under static loading conditions.

Nevertheless, the departure from the "more is better" approach to frame stiffness (first stage), and a general trend towards the beneficial use of elastic properties of frame materials (second stage) represents a significant step forward. According to this revised understanding, the stiffness of a bicycle frame cannot be represented by a line as shown in Figure 1, but it must be defined as a two-dimensional continuum as illustrated in Figure 2. An ideal frame would be laterally stiff and vertically compliant, pulling it towards the upper right corner of the graph.

But in practice, a 3-dimensional tubular structure that is stiff in one direction tends to be stiff in other directions as well. Therefore, an inverse relationship generally exists between vertical compliance and lateral stiffness (although, anisotropic properties - different properties in different directions - of carbon fiber composites allow individual frame designs to deviate from this general trend). This relationship is represented by a conceptual and hypothetical trend line (dashed curve) in Figure 2. If we took the four bikes that were characterized in Figure 1 and plotted their vertical and lateral measurements separately on orthogonal axes, a graph that is close to Figure 2 may emerge, which could potentially reveal additional information. For example, while Bike 2 and Bike 3 are relatively close to the sweet spot in Figure 1, Figure 2 reveals that Bike 3 actually stands out, as it combines above-average lateral stiffness with above-average vertical compliance. This concept of directional stiffness represents the *third stage* of understanding frame stiffness.

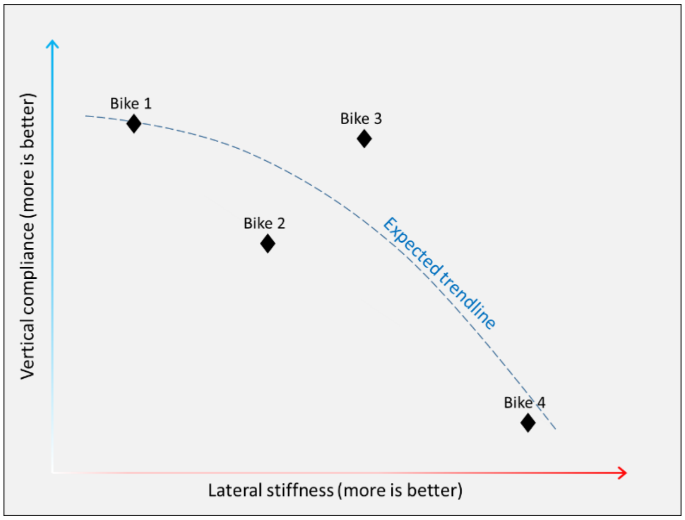


Figure 2 –Two dimensional visualization of frame stiffness

Local stiffness: The separation of frame stiffness into lateral and vertical components was certainly more informative than the one-dimensional stiffness concept, which, in turn, was more informative than the "more is better" concept that it replaced. But the vertical and lateral categories are still somewhat arbitrary and not entirely relevant, because the real-world forces acting on a bicycle frame are not constrained to those two directions. Forces and deformations in the fore-aft direction are also significant, and when rotational (pitch, roll and yaw) movements are also included, each point on the structure has, in general, 6 degrees of freedom. Furthermore, in practice, input forces are applied at numerous locations, each resulting in deformations in numerous other locations. Thus, no single location (or ratio) exists such that lateral and vertical measurements can represent the entire frame. In recognition of these facts, *typical* load conditions and deformation measurements have been defined, leading to application-specific stiffness values such as "bottom bracket stiffness", "head tube stiffness", "handlebar compliance", etc. This concept represents the *fourth stage* of understanding frame stiffness. Damon Rinard's measurements of frame and fork deflections are good examples of these efforts.

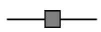
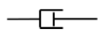
DYNAMIC STIFFNESS (MECHANICAL IMPEDANCE)

The term "stiffness" refers to a static loading of a structure, and is defined as the ratio between an applied *force* (input) and the resulting *displacement* (output). Similarly, torsional stiffness refers to twisting deformations where input and output are defined as torque and rotation, respectively.

But sometimes the load is not static. For example, when a person rides a bicycle on an uneven road surface, the load condition is dynamic (i.e., variable in time). When the magnitude and direction of an applied force are changing rapidly enough (i.e., under dynamic load conditions), the response of the structure depends not only on the instantaneous values of these quantities, but also on how quickly they are changing. This results in a frequency-dependent response function, in which case the static definition of stiffness falls short of fully describing the input / output relationship. The engineering concept of *mechanical impedance* is defined to address this shortcoming. For periodic forces such as those from pedaling, impedance is similar to stiffness except that the input / output ratio and phase depend on the magnitude *and frequency* of the input. When simulating the response of a structure to a dynamic input, substituting impedance for stiffness (i.e., using a dynamic model instead of a static one) allows the model to account for the time delay that the output lags behind the input. For vibrating structures, this delay (and thus phase) is a critical parameter for quantifying the behavior of the structure and analyzing a particular mechanism of energy dissipation known as *damping*. Keeping track of the response delay also allows the model to differentiate between reactive loads that don't dissipate power and resistive loads that do. This behavior of vibrating mechanical structures can be understood by means of a dynamic model comprised of lumped elements.

LUMPED-ELEMENT MODELS

The dynamic model of a structure can be thought of a circuit comprised of rudimentary components called *lumped elements*. For the model to be accurate, a large number of elements may be needed, but there are only 3 *types* of elements. Each element is modeled as a spring, a mass, or a damper.

- Spring 
- Mass 
- Damper 

These elements are differentiated by the nature of the pushback force with which they respond to mechanical inputs. When a *spring* is flexed or extended, the pushback force is proportional to the *displacement* of the flexion / extension (movement of one end relative to the other), and the ratio between force and displacement (stiffness) is denoted "k". When a *mass* is moved, the pushback force is proportional to *acceleration*, and the ratio between force and acceleration (mass) is denoted "m". When a *damper* (*aka* dashpot, shock absorber or 'shock' for short) is flexed or extended, the pushback force is proportional to the *velocity* of the flexion / extension, and the ratio between force and velocity (dashpot constant) is denoted "c". Springs and masses don't dissipate energy; consequently, suspension losses and rolling resistance occur only in dampers, but the velocity of each damper, and thus the amount of the energy loss in it, depend on the mutual interactions between *all* of the interconnected components in the system, whether they dissipate energy or not.

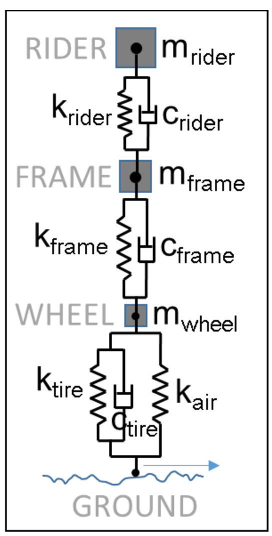


Figure 3 –3-DOF lumped element model of a person riding a bicycle on an uneven surface

In general, the potential for the accuracy of the model increases with the minimum number of independent displacements that are necessary to fully describe all possible deformations, which is called the number of Degrees Of Freedom (DOF). A sufficiently accurate dynamic model of a complex structure such as a person riding a bicycle on an uneven road surface would typically have dozens of degrees of freedom. But for the purpose of concept illustration, Figure 3 can be imagined as the schematic diagram of a simplified approximate model with only 3 degrees of freedom. Each degree of freedom is associated with the vertical displacement of one of the masses from its static equilibrium position. There are 3 masses, hence 3 degrees of freedom.

This overly simplified model doesn't even include two separate wheels. Instead, the combined mass of both wheels is lumped at a single point, and denoted $m_{\mathbf{wheel}}$. This mass is suspended from the ground by a mechanical circuit representing the viscoelastic behavior of pneumatic tires. The inflated tire (the sub-circuit inserted between the wheel and the ground in the schematic) is represented by a parallel connection of 3 elements: an air spring (compressed air), a rubber spring (tire casing) and a damper. The spring constants are $k_{\mathbf{tire}}$ for the rubber spring and $k_{\mathbf{air}}$ for the air spring. The damper with the dashpot constant $c_{\mathbf{tire}}$ represents the damping mechanism due to the energy-dissipating nature of the material the tire is made of, but the compressed air is idealized as a lossless spring. While $k_{\mathbf{tire}}$ and $c_{\mathbf{tire}}$ are associated with geometric and material properties of the tires, the value of $k_{\mathbf{air}}$ can be adjusted by changing the tire inflation pressure. The combined mass of the two wheels ($m_{\mathbf{wheel}}$) is connected to the mass of the frame ($m_{\mathbf{frame}}$) via a spring element whose stiffness is denoted $k_{\mathbf{frame}}$. This spring element represents the combined elasticity of the frame and fork.

The model, which is barely adequate for a unicycle, is constrained by the above approximations and assumptions in order to make the number of degrees of freedom as small as possible, in an attempt to explain by inference how a full-grown lumped element model could be applied to vibration problems.

Model construction: As the bicycle travels from left to right as indicated by the arrow, the uneven profile of the road surface induces movements and reaction forces in each of the elements in the circuit. For a given road profile and bicycle speed, with *how much* force and movement each element responds to the forces and movements of other elements in the circuit is defined by the circuit diagram and universal equations of motion. Mechanical-electrical analogies allow us to set up and solve these equations using computer software developed for analyzing AC electrical circuits. Simulation results in this paper are obtained with this method using circuit design software called LTspice® by Analog Devices™.

Model validation: The mass, stiffness and damping values of all lumped elements in the simulation model must be "tweaked" until it is demonstrated that the vibration response of the model corresponds to that of the actual structure measured in a variety of load cases and conditions. This means that the model is validated by test data.

Simulations: Once a model is validated, the circuit diagram defining the lumped elements and their interconnections remains unchanged, but the mass, stiffness and damping values of the elements can be changed at will. Road profiles can also be defined arbitrarily. Each simulation with a new set of values is equivalent to having evaluated a new design iteration, but without having to modify, build and test a new physical structure. This practice dramatically simplifies and shortens the product development cycle. As well, unexpected findings revealed by simulating unusual "what if" scenarios often lead to new insights about relationships between design variables that would otherwise not be discovered.

SPECIAL CASES

Sometimes the general behavior of a circuit can be better understood or explained by evaluating its performance in special scenarios. Below are a few such scenarios which may be useful for constructing a better developed mental image of the model.

Figure 3 allows us to visualize that if the bicycle speed is sufficiently slow and if the spring constants are sufficiently high, all 3 masses move up and down in lock-step to follow the road profile faithfully (alas, with a small smoothing effect due to the finite size of the tire contact patch, but our model assumes that the dimensions of the contact patch are small relative to the bumps). In other words, the displacement amplitudes of the vibrating masses will be nearly equal to the vertical height of the bumps on the road. This is certainly true at extremely low speeds. But in the opposite end of the speed spectrum where the bicycle speed is sufficiently high, as the tire transmits the bump force to the rim, the rim pushes back with a large reaction force due to the inertia of the masses attached to it, causing the tire to deform and soak up the bumps (provided that the bumps are not taller than the height of the unloaded tire). Consequently, at extremely high speeds, when the wheel moves over small bumps (i.e., bumps that are not too tall), the mass elements follow a smooth line that is parallel to the ground, without vibrating. Thus, we can see that at low speeds the roughness of the road is substantially transmitted to the rider, while at high speeds it is significantly attenuated by the tire.

For this “thought experiment” we have considered the road to be unchanged, and varied the speed. But we could have just as well kept the speed constant and varied the length of the bumps. It should be clear that the degree to which road shock is absorbed in the tire depends neither exclusively on the bicycle’s speed nor exclusively on the bump length, but rather, it depends on the *frequency* at which the tire contact patch vibrates in the vertical direction (base excitation). It is just that this excitation frequency is equal to the ratio between the speed of the bicycle and the wavelength of the bump. The low frequency components of the input vibrations are transmitted to the rim, but high frequency components are absorbed in the tire. Thus, the structure is said to function as a lowpass filter (LPF).

Resonance: In general each bump has a different length, and the bike speed is not constant either. Therefore, the dynamic force directed upward from the contact patch on the ground towards the wheel hub is comprised of many vibration components distributed across a *spectrum* of frequencies. We have seen that low-frequency components of the vibration are transmitted, but high-frequency components are filetered out by the tire. Now, we investigate what happens at intermediate frequencies.

Behavior of dynamic systems is governed by a phenomenon called *resonance*. Each resonance of a structure is characterized by a unique natural frequency and mode shape. A physical structure has as many natural frequencies as it has degrees of freedom. Although the simplified model shown in Figure 3 has only three natural frequencies, a person riding a bicycle has many more. When a frequency of excitation coincides with a natural frequency of the structure, resonance occurs. Vibration input components with characteristic frequencies that are much less than the lowest natural frequency are transmitted rather faithfully, and those at frequencies much greater than the highest natural frequency are hardly transmitted at all.

Multi degrees of freedom: When a structure has a large number of natural frequencies, a given input frequency will be, in general, greater than some of the natural frequencies but less than others. In general, the vibration input (stimulus) also has multiple frequency components. Thus, vibration inputs will be partly transmitted and partly filtered, depending on the location of stimulus components on the frequency spectrum relative to the various natural frequencies of the structure. Thus, a multi-degree-of-freedom structure also functions as a vibration filter, but it cannot be characterized as a simple LPF. Rather, its filter characteristic, which is a function of the distribution of its natural frequencies, is called the *frequency response* of the structure. A typical frequency response contains peaks and dips. For example in Figure 4, the frequency response of the wheel has two peaks and a dip in the middle; the frequency responses of the frame and rider have only one peak at nearly the same frequency.

Damping: At the frequency of resonance, the vibration response of the system is governed by *damping*. Damping means that a portion of the vibration energy is dissipated as heat, causing the vibration amplitudes to change. A change in amplitude doesn't necessarily mean a reduction in amplitude. When the vibration input is base excitation, at or below a natural frequency, the vibration amplitudes of damped degrees of freedom will be less than they would be in the absence of damping, but at much higher frequencies they are greater. Underdamped degrees of freedom have a high transmissibility of vibrations near the frequency of resonance, but overdamped degrees of freedom have a high transmissibility at high frequencies. Thus, critical damping is generally sought for all degrees of freedom, but for special applications there may be exceptions to this rule. Critical damping is the amount of damping that allows a displaced degree of freedom to return to its original position as quickly as possible.

As far as road shock is concerned, vibration damping on road bikes is provided to a large extent by the viscoelastic (lossy) nature of the tire material. But some damping also occurs in other parts of the structure due to deformations of the frame and fork, and in the soft tissues of the rider. This reflects the energy-dissipating nature of muscles, ligaments and tendons; friction forces between internal organs that are jostled due to vibration also provide damping; friction due to loosely fitting clothes, accessories, items carried in panniers, seat bags or pockets, etc. In the simplified model depicted in Figure 3, all of the damping elements in the human body are lumped together and represented by a single dashpot constant $c_{\mathbf{rider}}$, but the full model would include numerous dampers, each one having a separate dashpot constant.

Accuracy of the model: For the dynamic simulation model to be accurate enough to be of practical value, the circuit would have to be comprised of a much bigger and more elaborately interconnected network of lumped elements than the simple 3-degree-of-freedom circuit shown in Figure 3. Nevertheless, we will use this simple model to demonstrate the value and efficacy of the dynamic simulation approach, with the understanding that this particular model, which is too basic to be of practical use, can potentially be developed into a full model.

Parameter values: The baseline values of the 3 lumped masses are chosen to be 70kg for the rider, 8kg for the frame including components and accessories, and 2kg for the wheels.

The baseline values of the spring constants representing the inflated tire and the stiffness of the bicycle frame are taken from the following source:
https://cyclingtips.com/2018/04/jra-with-the-angry-asian-does-frame-compliance-still-matter/

The baseline value of the spring constant representing the stiffness of the rider's body is chosen to be consistent with a 70kg person's having a 5Hz frequency of resonance, as described in the following source: https://www.hindawi.com/journals/js/2018/7140610/

SIMULATION RESULTS

The model allows us to simulate different kinds of scenarios that may be of interest. Depending on the nature of the simulation, it is convenient to present the results in either the *frequency domain* or the *time domain*. Although these are two alternate ways of looking at the same thing, we will explain these scenarios separately.

FREQUENCY RESPONSE:

As far as vibration control is concerned, one of the most important characteristics of a bicycle is its frequency response.

Figure 4 is the frequency response to base excitation of the 3-degree-of-freedom lumped-element simulation model shown in Figure 3.

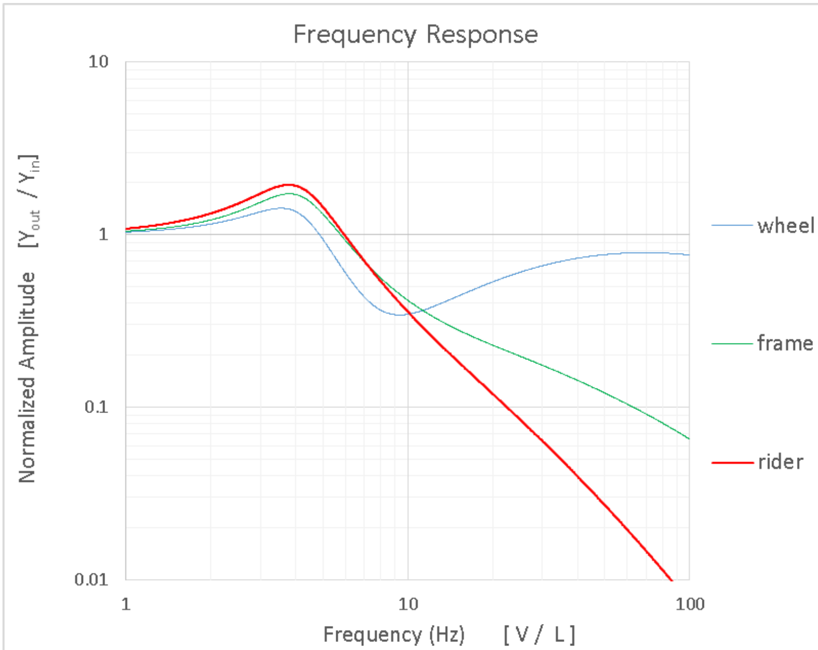


Figure 4 –Simulation result showing the baseline frequency response when the bicycle is ridden on a rough road. Normalized amplitude is the vertical displacement amplitude of the selected part relative to the roughness of the road.

The simulated road can be thought of as comprised of many perturbations (bumps) that are randomly distributed in height and length (base excitation with random noise). The quantity plotted on the vertical axis, normalized amplitude (Y_{norm}), is then the root-mean-square (rms) amplitude of the vertical displacement of the selected component (Y_{out}) divided by the root-mean-square height of the perturbations on the ground (Y_{in}) which we may refer to as the *roughness* of the road.

Normalized Amplitude $Y_{norm} = Y_{out} / Y_{in}$.

In this model, the lengths of the perturbations are assumed to be much greater than the effective length of the contact patch, so that the contact patch functions as a simple stylus that follows the instantaneous vertical coordinate of the perturbation as the bike is ridden over it. This assumption compromises the model's accuracy on roads where the perturbations are short, such as small cracks and pebbles on an otherwise smooth road. It is also assumed that the contact patch is narrower than the width of the perturbations. In future revisions of the model, these limiting assumptions will be removed.

The quantity plotted on the horizontal axis of Figure 4 is the vibration input frequency in Hz. It is equal to the ratio between the bicycle speed (V) and the length of the perturbation (L).

Excitation frequency $f = V / L$.

Therefore, the horizontal axis could just as well be labeled “normalized speed”. For example ,if one rides a bicycle at a speed of 1 meter per second over bumps that are 1 centimeter in length (…or more generally at ‘n’ meters per second over bumps that are ‘n’ cm long), the excitation frequency (or the quantity we may call normalized speed) is 100Hz.

In other words, the horizontal axis shows what fraction of a second it takes to traverse a bump. Low speeds (and/or long bumps) plot to the left of the axis; high speeds (and/or short bumps) plot to the right. It is as if the tire contact patch is vibrating vertically at a given displacement amplitude and frequency, and Figure 4 allows us to see the vertical amplitudes of the 3 degrees of freedom (namely, the masses associated with the wheel, frame, and rider).

At frequencies where the output amplitude is less than the input amplitude (i.e., where the normalized amplitude value is less than 1.0), road vibration is attenuated. At frequencies where the normalized value is greater than 1.0, it is amplified. Logarithmic scales are chosen for both axes to show a wide range of values on the same graph without giving up too much resolution.

As shown in Figure 4, at sufficiently slow speeds (or on sufficiently long bumps - namely at or below 1Hz), the normalized amplitudes of all 3 degrees of freedom approach 1.0, meaning that all three masses ($m_{\mathbf{wheel}}$, $m_{\mathbf{frame}}$ $m_{\mathbf{rider}}$) accurately follow the bumps. It is noted that at input frequencies between 1Hz and 6Hz, road bumps are amplified (dynamic amplification of base excitation), indicative of one or more underdamped degrees of freedom. At higher speeds and/or shorter bumps (i.e., higher frequencies), the general trend of a low pass filter is evident for the vibration of the rider (red curve).

If our model were accurate, the shape of this frequency response would be a strong indicator of the ride quality of the bicycle. But to solve a vibration problem as complex as this one, a model with only three degrees of freedom cannot be accurate enough to lead to meaningful conclusions. Furthermore, component values including damping coefficients must be determined by testing real people and bicycles rather than arbitrarily assigning estimated values to demonstrate a concept, as I have done. Nevertheless, I ran a few simulations using this model to demonstrate the utility of constructing and validating a full and accurate model. We are interested only in the red curves (vibration of the rider), but the vibrations of the wheels and frame are also given for additional information.

By changing the mass, stiffness and damping values of individual components in the model and observing the simulated outcome of each modification, we can gain insight and intuition about which design factors are more relevant than others, and how they may affect ride quality.

The full model will have many variables to optimize, but our simple model has only a few. Below we look at the effects of three of them, namely, tire pressure, frame stiffness, and riding position.

- <u>Tire Pressure</u>: In our model, the value of the parameter $k_{\mathbf{air}}$ represents the radial stiffness of the wheel on the riding surface, which is controlled by the tire inflation pressure. We assume that for a given tire/rim combination, within the range of normal operating pressures (60 – 120 psi or 4 – 8 bar for a road-racing bike) $k_{\mathbf{air}}$ is directly proportional to the tire pressure.
- <u>Frame stiffness</u>: As stated before, to characterize all of the relevant elastic properties of a bicycle frame fully, we would need a large number of parameters. These might include variables such as head-tube angle, down-tube diameter, chainstay length, fiber layup, and other engineering variables. These are difficult to change on an actual frame, because for each change a new frame must be made. But in our lumped element model, the values of all spring, mass and damping elements are contained in a table of numbers which can be easily changed at will. In

our approximate 3-degree-of-freedom model, the entire table of numbers for all of the spring elements in the frame and fork is represented by a single number, $k_{\mathbf{frame}}$.

- Riding position: If the number of variables to specify the dynamic properties of a bicycle frame is large, then the number for the human body must be even larger. But the riding position is no doubt one of those variables. For example, holding the handlebar with extended arms makes for a relatively stiff connection between the frame and the rider's shoulders, while bent elbows with relaxed muscles make for a more compliant connection. The frame and the rider's body are connected through other paths as well, but in our model, the stiffness of the entire network of connections between the two masses $m_{\mathbf{rider}}$ and $m_{\mathbf{frame}}$ is defined by the value of a single parameter, namely, the spring constant $k_{\mathbf{rider}}$.

We can change the values of these parameters one at a time, run the simulation and learn from the results. Or we can do a "design of experiments" if we want to minimize the number of iterations needed for checking all possible interactions between parameters.

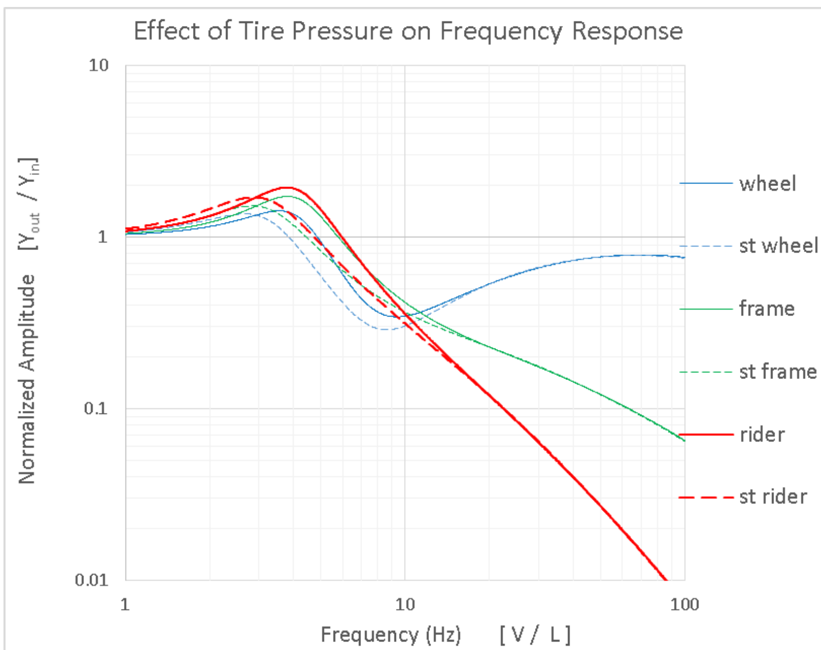


Figure 5 –The effect of lowering the tire inflation pressure by 50%. Traces labeled with the "st" prefix in the legend indicate the simulation with the low pressure ("soft tire") modification.

Figure 5 shows the effect of lowering the *tire inflation pressure* by 50% compared to its baseline value. This modification improves the rider's vibration isolation on relatively long bumps / slow speeds (i.e., when the input frequency is between 3Hz and 15Hz), but we see no significant effect at lower or higher frequencies. Notably, above 20Hz there is no effect on the vibrations of the wheel and frame, either. It should be noted that low tire pressure has the additional benefit of increasing the area of the tire's contact patch with the ground (short bumps are averaged out regardless of bicycle speed), but that effect is not included in this version of the simulation model.

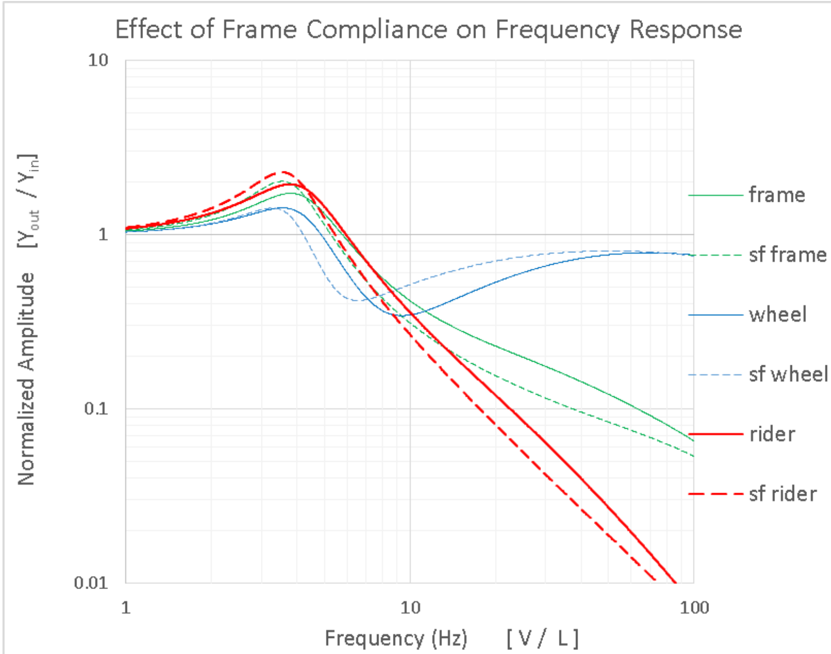


Figure 6 –The effect of reducing frame stiffness by 50%. Traces labeled with the "sf" prefixes in the legend indicate the simulation with the "soft frame" modification. For the rider, vibration attenuation is significantly improved between 4.5Hz and 100Hz. On the other hand, vibration amplification is increased between 1Hz and 4.5Hz.

In Figure 6 we see the effect of *frame stiffness*. The simulation model suggests that a more compliant frame would increase the vibration isolation of the rider within a broad range of bump lengths / speeds (covering a frequency range between 4.5Hz and 100Hz), but the more compliant fame makes things *worse* around 3.5Hz.

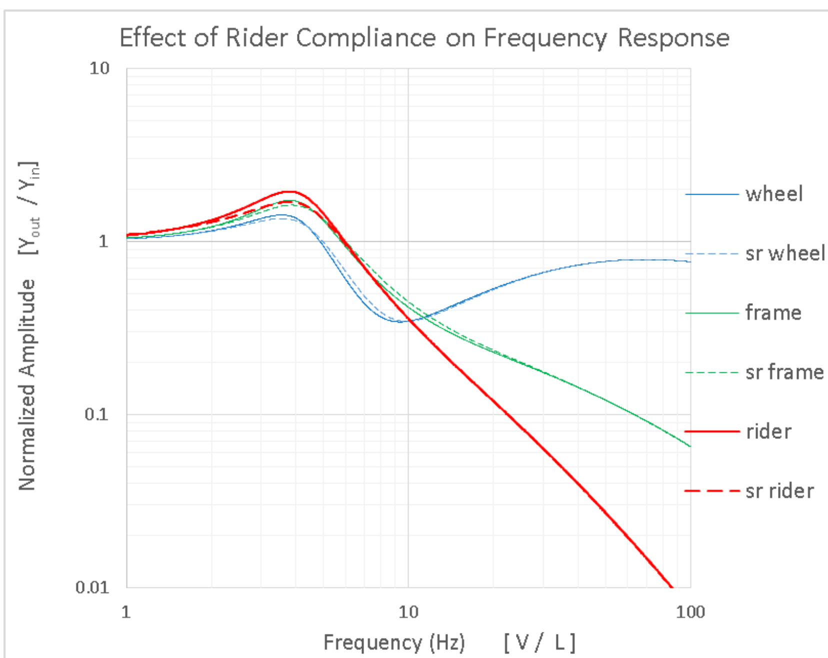


Figure 7 –The effect of changing the riding position. The "sr" prefixes in the legend indicate the simulations with the "soft rider" modification. When the rider's body stiffness is reduced by 50% relative to the baseline, vibration attenuation is slightly improved within a narrow frequency band (2Hz - 5Hz).

In Figure 7, we see the effect of reducing the spring constant of the lumped element representing the stiffness of the rider's body by 50%. This corresponds to modifying the riding technique, for example by holding the handlebar with bent elbows rather than straight arms as alluded to earlier. Another technique to make the connection between the rider and frame more compliant might be to select a high gear ratio or to accelerate (or even apply downward pressure to both pedals at once) when riding over bumps, so that the effective spring between the rider's hips and the frame's bottom bracket is defined by a flexible linkage mechanism involving the femur, knee, shin, ankle, foot, pedal and crank, rather than by the more direct and rigid path through the saddle, seat post and seat tube. Certain changes in the effective stiffness of the rider's body will necessarily result in the transfer of some mass from the rider to the frame which then also needs to be modeled. In our model with only three degrees of freedom that doesn't simulate that effect, the spring constant $k_{\mathbf{rider}}$ affects the vibration amplitude of the rider's body only in a narrow band of frequencies (between 2Hz and 5Hz in this case), and the improvement is rather small.

TRANSIENT RESPONSE:

So far, we have simulated the system's response to *steady state* vibration inputs, and interpreted the results in the *frequency domain*. But as stated earlier, once we have a validated model, we can also simulate *transient* inputs and interpret the results in the *time domain*.

For example, when the bicycle is ridden over an obstacle (such as a surface crack, expansion slot, speed bump, pothole, etc.), we may want to know how each of the three lumped elements (wheel, frame and rider) would move. Our aim is to "tune" the parameters of the model to reach a desired ride quality. If our model is accurate, and if we can correctly identify the desired qualities, and if we can successfully transfer the virtual modifications to an actual bicycle, then, the process of parameter optimization or "tuning" with the help of a dynamic simulation model might be an important engineering tool for designing road bicycles.

First, we'll simulate the bicycle being ridden over the rumble block shown in Figure 8.

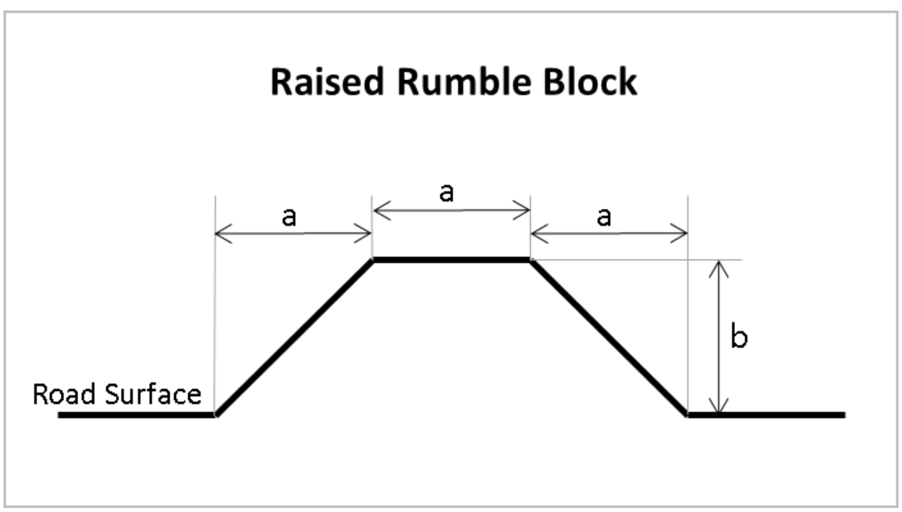


Figure 8 –Elevation profile of a raised rumble block

In the simulation model, the block is used in such a way that dimension 'b' defines the measurement unit of vertical movement, and dimension 'a' defines the ratio between bicycle speed and input frequency. We neglect the curvature of the wheel in this version of the model. The tire contact patch is modeled as a simple probe that samples the instantaneous height of the block above the reference road surface.

It is clear that if the bicycle's speed were sufficiently low (e.g., potentially even slower than walking speed), all three lumped masses in our model would move in lockstep, exactly following the elevation profile of the block. But at a sufficiently high speed, the interactions between the masses and the flexibility of the tires, frame and the rider's body will come into play, and consequently, each mass will trace a different path as the bike is ridden over the block.

Our primary interest is knowing the path of the rider's body, but in order to get that information, our model must solve simultaneous equations that include all three masses as well as all of the springs and dampers. Figure 9 shows the solution of these equations for three different speeds.

Figure 9 –Transient response. The bicycle is ridden from left to right. The elevation of the road profile (projected to the secondary axis on the right) is scaled such that the dimension b of the raised block specified in Figure 8 defines the unit of elevation. The red trace (projected onto the primary axis on the left) is the ratio between the vertical displacement of the rider's body from its static height above the ground and the overall height (dimension b) of the block. The green and blue traces represent the same quantity for the wheel and frame, respectively.

The horizontal axis represents the distance travelled on the ground. The dimensionless quantity on the primary (left) vertical axis is the ratio between the vertical displacement of each mass from its static position, and the overall height of the bump (dimension b in Figure 8). Because the road profile is also normalized with respect to dimension b, the rumble block is 1 unit tall by definition.

The rationale for normalizing the vertical axes but not the horizontal axis is that the solution is specific to the shape of the bump but agnostic about the height of the bump. For a given speed, the length of the bump defines the *frequency* of the input and thus engages the structure according to its frequency response, whereas the height of the bump determines only the *amplitude* of the input, to which the structure's response is proportional (within elastic limits).

For example, if the bicycle is ridden over a book lying on the ground, the waveform defining the path of the rider relative to the ground will depend on the thickness of the book, but the vertical aspect of the waveform will also scale with the thickness of the book, so that the *normalized waveform* will be independent of the thickness of the book. But riding over the same book at different speeds would produce entirely different waveforms. On the other hand, riding over a book that is twice as large (in length) would elicit the same normalized response as riding at half the speed. The variables in Figure 9 are normalized such that the traces are drawn as they would appear to an observer on the ground, and the solution is applicable to the full set of raised blocks conforming to Figure 8 regardless of the a/b aspect ratio.

Interpreting Figure 9, we can make several observations about the response of the system to transient inputs arising from riding over this obstacle at these speeds.

- At the slowest speed simulated (3.1m/s or 7mph), the maximum vertical displacement of the rider's body is approximately 120% of the height of the block (normalized peak value is approaching 1.2), and with increasing speeds, it becomes progressively smaller.
- At the slowest speed the wheel and frame move more or less together, whereas at the highest speed (12.5 m/s or 28 mph) they follow significantly different paths, as the peak amplitude of the rider diminishes and that of the wheel increases with speed. Note also that the frame and rider follow the wheel's lead by a lag that increases with speed.
- When riding over this particular obstacle at these particular speeds, some of the "suspension" function appears to come from the rider's body. So it makes sense to check to see if the riding position makes a big difference.
    - To find out, we look at the effect of doubling the compliance of each of the spring elements in our model (i.e., we halve the values of k_{air}, k_{frame} and k_{rider}) one at a time while holding the other two at their baseline values, just as we did for frequency response. When we do that for the highest speed (12.5 m/s or 28 mph), the model predicts the following changes in the peak vertical amplitude of the mass center of the rider's body:

parameter	stiffness reduction	vibration reduction
k_{air}	50%	21%
k_{frame}	50%	3%
k_{rider}	50%	7%

    - In short, our model indicates that when riding over this bump at this speed, lowering the tire pressure would be by far the most effective of the three improvement options, if the goal were to minimize the peak vibration amplitude of the mass center of the rider's body.

Our model is not accurate enough to extend these conclusions to an actual bicycle, for two main reasons. One ris that the model has only three degrees of freedom, not nearly enough to accurately simulate a system as complex as this one. This issue could be addressed by adding more elements to the model. The second reason is that the parameter values we use in the existing model are rough estimates at best. For reasonable accuracy, these values must be obtained by measuring the actual structure being simulated, requiring the development of equipment and procedures for testing in the lab and in the field.

Nevertheless, these simulation results demonstrate the value of employing lumped-element dynamic modeling techniques to the designing of frames and forks for road bicycles. If nothing else, these results highlight the importance of incorporating in the model, copious amounts of accurate information about the mass distribution and elastic / viscoelastic properties of the rider's body. To be reliable, the simulation model must include many degrees of freedom, and the network of lumped elements must be intelligently laid out so that the circuit diagram represents the actual structure accurately. In addition, the parameter values in the model must be determined through reliable testing.

The importance of mass distribution: As crude as this model may be, it allows us to investigate the effect of removing some mass from the bicycle and adding it to the rider, all other parameters being equal. I have been curious about this question for some time; so in the model I reduced the values of $m_{\mathbf{wheel}}$ and $m_{\mathbf{frame}}$ by 50% each, and increased the value of $m_{\mathbf{rider}}$ by the same amount (not by 50%, but by the total amount removed from the wheels and frame). Then I simulated the case of the heavier cyclist riding the lighter bike over the same obstacle at the same three speeds as before. This modification corresponds to carrying some weight in a jersey pocket or a backpack instead of in a seat bag or panniers, for example. Lo and behold, the model predicts that the peak vibration amplitude of the bicycle will increase, but that of the rider will decrease. The predicted amount of reduction in peak amplitude of the rider is 3.2%, 3.9**%** and 4.0**%** for the simulated slow, medium and high speeds, respectively.

Riding on a rumble strip: Thus far, we have performed steady -state and transient simulations, and interpreted the results in the frequency domain and time domain as needed. But sometimes the actual situation may be neither purely steady-state nor purely transient. It may be somewhere in-between, or a combination of both. Riding temporarily across a rumble strip next to the "fog line" on a highway is a good example of this situation.

Typically, highway rumble strips are milled in order to accommodate snow removal, but we are simulating a strip of rumble blocks that are raised above the road surface. At the onset of contact, raised rumbles elicit a different response than milled ones, but due to the constant spacing between the blocks in the strip, steady-state response characteristics eventually come into play whether the rumbles are raised or milled. For example, in Figure 10 we observe that the system response reaches a steady state at approximately 1.5 meters at the slowest speed, but at the higher speeds, the rumbles end before a steady state is reached. Transient behavior is also observed immediately after exiting the rumble strip. So this scenario is an example where we can observe both transient and steady-state behaviors of the system.

Cyclists experienced in riding across rumble strips or other periodic surface perturbations such as cattle guards, open-grate bridges, etc. are familiar with the phenomenon that if you enter the obstacle at the wrong speed (for example going slowly on a climb) the vibration can be jarring, whereas at a sufficiently high speed (for example going downhill) the bicycle seems to glide smoothly over the same obstacle in relative comfort. Figure 10 is a good example of a simulation of this phenomenon. The red curve represents the trajectory of the center of mass of the rider's body, and we can see that the "wiggles" that are synchronized with the period of the rumbles (4a length units according to Figure 8) are substantially attenuated.

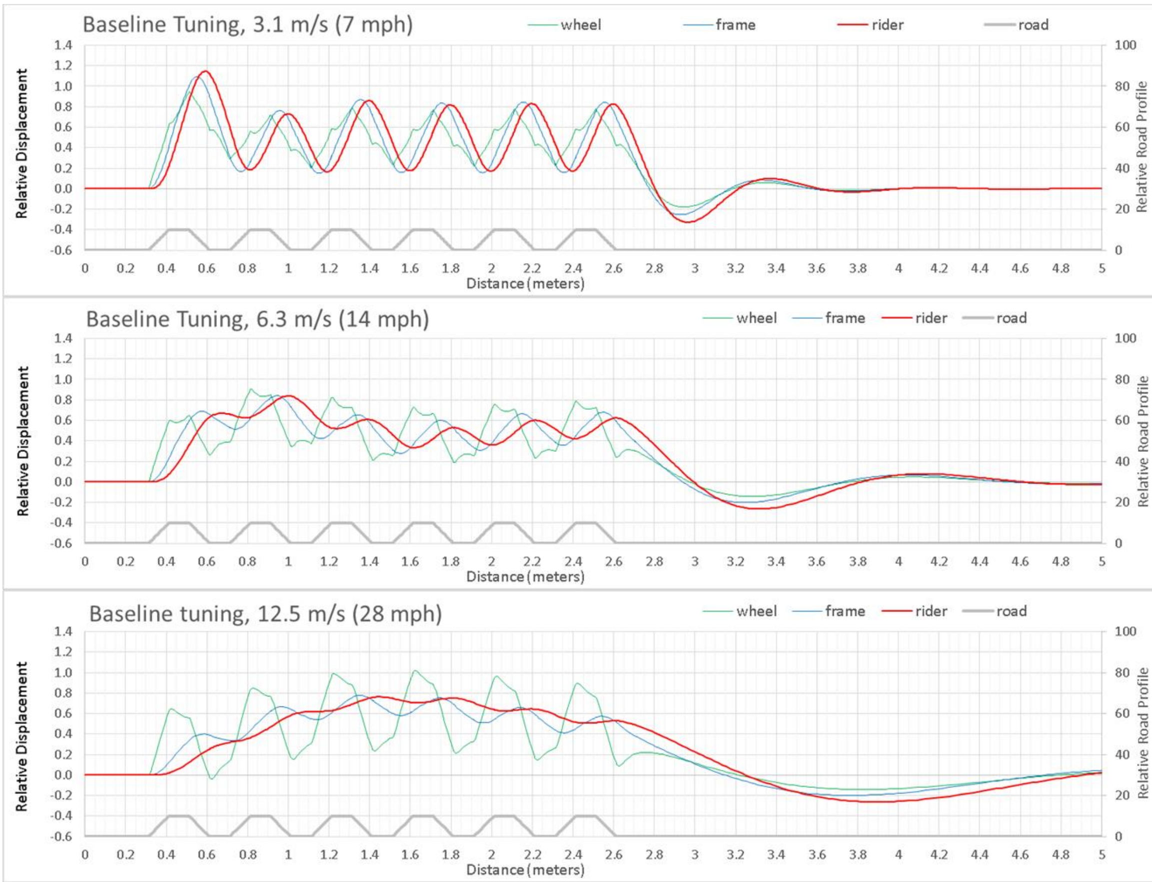


Figure 10 –Simulation of diagonally crossing a raised rumble strip.

Choice of output variables: The model allows us to investigate other variables that may correlate with rider discomfort. For example, in Figure 11 we simulate crossing the same rumble strip at the same three speeds shown in Figure 10, but rather than looking at the motion of each mass element, we look at the deformation of each spring element. The deformation of the spring representing the rider's body may be particularly relevant, but we plot the other two deformations for additional information.

Relative deformation (or relative flex) is defined as the ratio of the vibration-induced elongation of a spring element in the circuit to the height of the bump inducing the vibration. For example in Figure 11, frame flex is the instantaneous change in the distance between $m_{\mathbf{wheel}}$ and $m_{\mathbf{frame}}$, and relative frame flex is that quantity divided by the overall height of the rumble block (dimension 'b' in Figure 8). The *small-signal assumption* inherent in our simulation model holds that this ratio (i.e., relative deformation) is independent of dimension 'b'.

Comparing Figure 11 with Figure 10, it is noted that as far as the rider's body is concerned, the reduction of the elongation amplitudes with increasing speed (Figure 11) is not nearly as pronounced as the reduction of the motion amplitudes (Figure 10). The question whether the perception of discomfort is more closely related to the deformation amplitude or the motion amplitude is outside the scope of this analysis.

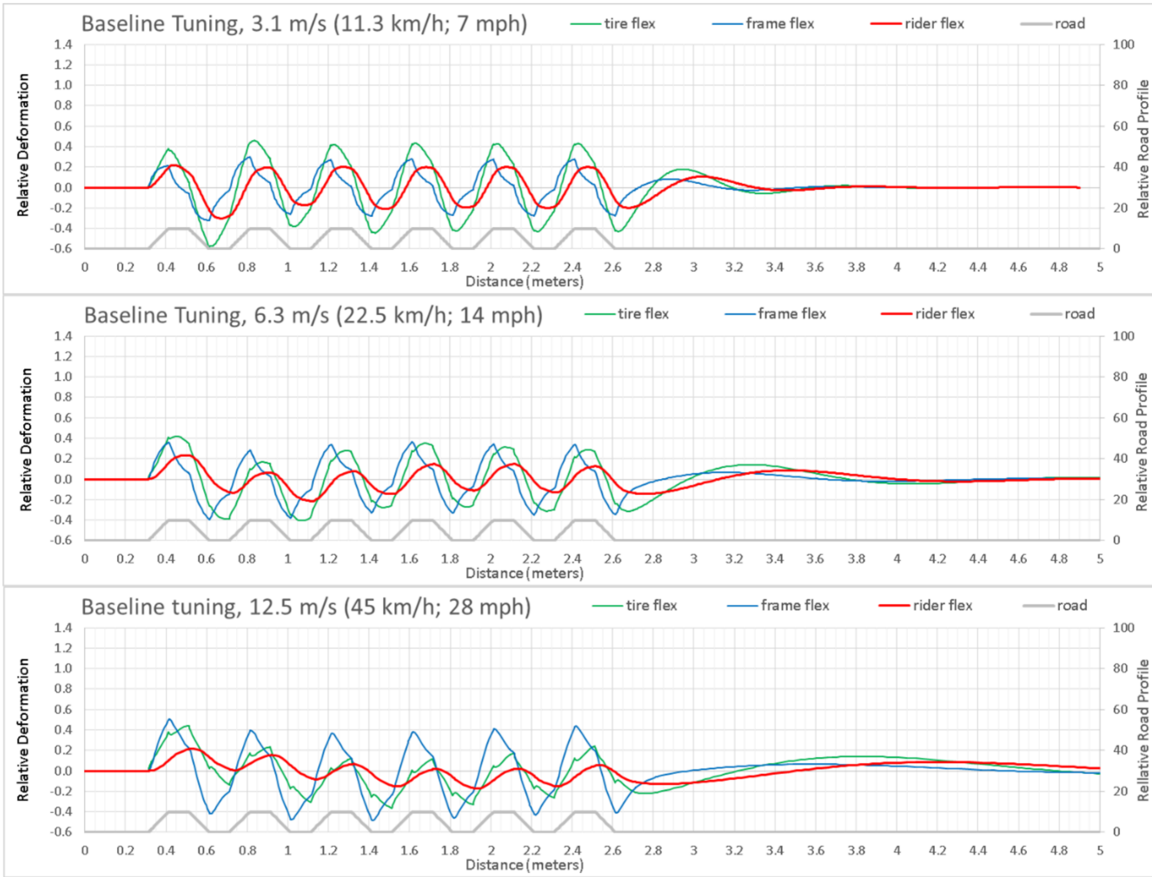


Figure 11 –Relative deformations of the tire, frame and rider

When interpreting the results of these simulations, it is important to keep in mind that this is not a perceptual model. We are not quantifying discomfort with these simulations; we are quantifying vibration. Even if we expect there to be a strong relationship between vibration and discomfort, we must be cognizant that our model does not include a simulation of that relationship. However, the output of our vibration model can provide valuable input to a perceptual model.

SIMULATIONS TO IMPROVE PERFORMANCE

This concludes Part One of this article, where our primary focus is on improving comfort. Dynamic simulation models can also be useful for improving *performance*, by helping us control a particular mechanism of energy dissipation called *suspension losses*, also known as *bump resistance*. In Part Two of the article, we shall investigate that mechanism. My plan is to update the existing dynamic model with additional degrees of freedom and new features, make the distinction between active and reactive loads (and consequently between true power and apparent power), explain how the choice of tires, frames and riding techniques can affect an engineering variable called Power Factor (in an analogy to AC electric power networks), define a parameter called *Dynamic Tire Rolling Resistance,* and show how to minimize it with the help of dynamic lumped element modeling.